Glacier Melt Under Debris

January 14, 2018

Kraaijenbrink et al. (2017) compiled data on the impact of debris cover on glacier melt (Fig. S5).

Full Reference: Kraaijenbrink, P. D. A., Bierkens, M. F. P., Lutz, A. F., & Immerzeel, W. W. (2017). Impact of a global temperature rise of 1.5 degrees Celsius on Asia's glaciers. Nature, 549(7671), 257–260. https://doi.org/10.1038/nature23878

The black line is the average of their findings. The purpose of this workbook is to identify an analytical function that approximates the black line.

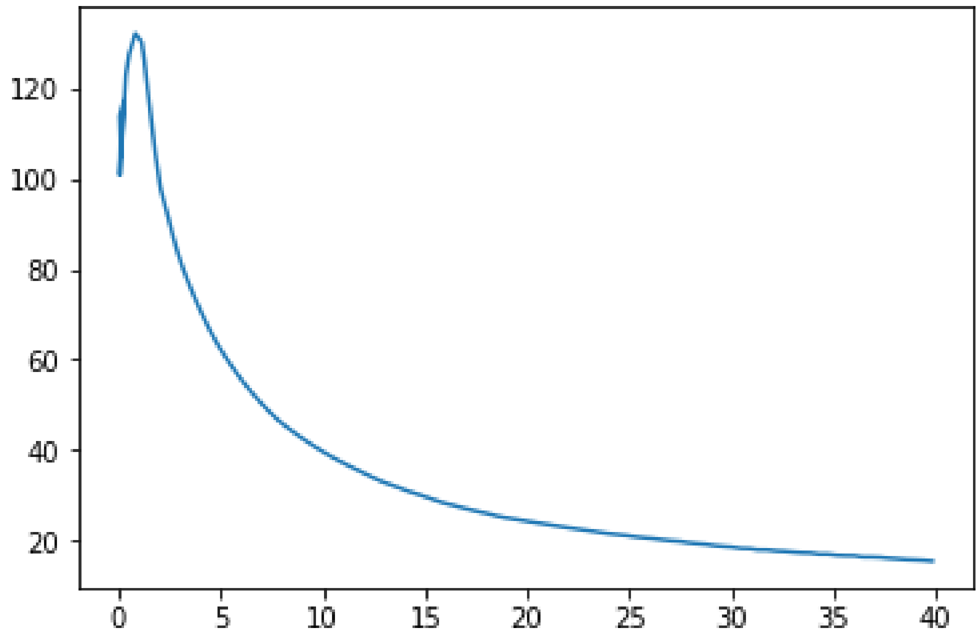
The first step in this process is using the data scraper https://apps.automeris.io/wpd/ The output is a CSV file of data points along the black line.

```
In [1]: import numpy as np

        from numpy import genfromtxt
        debris_data = genfromtxt('./data/debris_cover_melt_impact_Kraaijenbrink.csv', delimiter=
```

```
In [2]: x = debris_data[:,1]
        y_obs = debris_data[:,0]
```

Plot data:

```
In [3]: import matplotlib.pyplot as plt
        %matplotlib inline

        plt.plot(x, y_obs)

Out[3]: [<matplotlib.lines.Line2D at 0x10f6ba9b0>]
```



Check the maximum point (used in some of the function defs)

```
In [4]: indymax = np.argmax(y_obs)
        print(x[indymax], y_obs[indymax])
```

```
0.82 132.09
```

1 Implement three functions that can be optimized to fit the data

1.1 Define piecewise linear function that can be optimized to fit data:

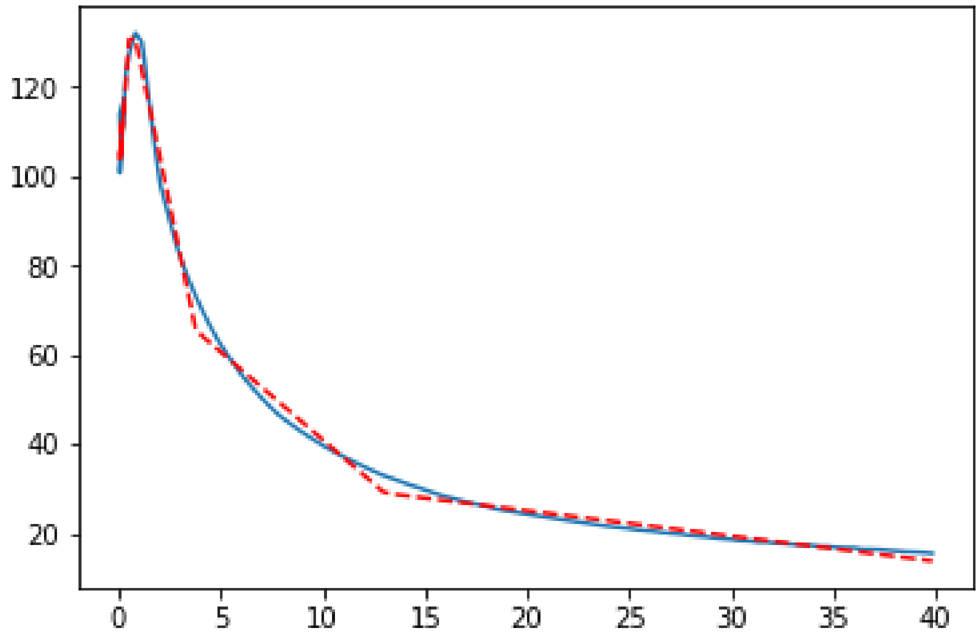
```
In [5]: def pwlin(x, a1, a2, a3, m1, m2, m3, m4):
            a0 = 0

            b1 = 100
            b2 = b1 + m1*(a1-a0)
            b3 = b2 + m2*(a2-a1)
            b4 = b3 + m3*(a3-a2)
            conds = [
                      (x >= a0) & (x < a1),
                      (x >= a1) & (x < a2),
                      (x >= a2) & (x < a3),
                       x >= a3
                    ]
            funcs = [
```

```
                lambda x: b1 + m1*(x - a0),
                lambda x: b2 + m2*(x - a1),
                lambda x: b3 + m3*(x - a2),
                lambda x: b4 + m4*(x - a3)
               ]
        return np.piecewise(x, conds, funcs)
```

```
In [6]: from scipy.optimize import curve_fit

        poptlin, pcovlin = curve_fit(pwlin, x, y_obs, p0 = [1.1,4,15,37,-18,-4,-0.8])
```

```
In [7]: plt.plot(x, y_obs)
        plt.plot(x, pwlin(x,poptlin[0],poptlin[1],poptlin[2],poptlin[3],poptlin[4],poptlin[5],po
        plt.show
```

```
Out[7]: <function matplotlib.pyplot.show>
```



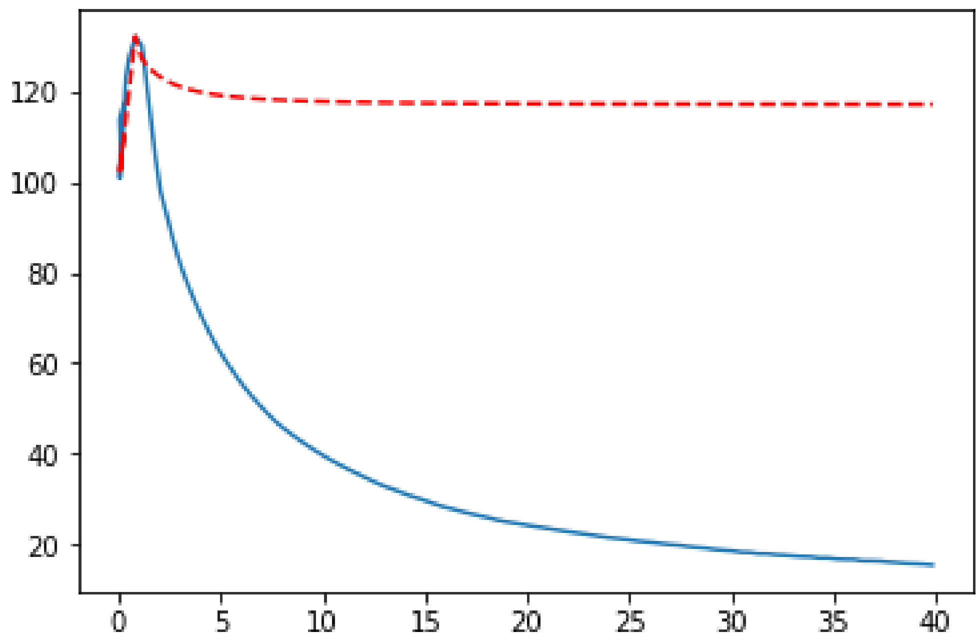
1.2 Define quadratic function to fit the data

```
In [8]: def pw1x(x, m2, c2, b2):
            y0 = 100
            y1 = 132.09
            a0 = 0
            a1 = 0.82

            m1 = (y1-y0)/(a1-a0)
```

```
    #m2 = max(m2,0.0001)
    c2 = max(c2,-a1+0.001)

    conds = [
              (x >= a0) & (x <= a1),
               x > a1
            ]
    funcs = [
              lambda x: y0 + m1*x,
              lambda x: y1 + m2/(x + c2)**2 + b2,
            ]
    return np.piecewise(x, conds, funcs)
```

In [9]:
```
plt.plot(x, y_obs)
plt.plot(x, pw1x(x, 100,2,-15), 'r--')
plt.show
```

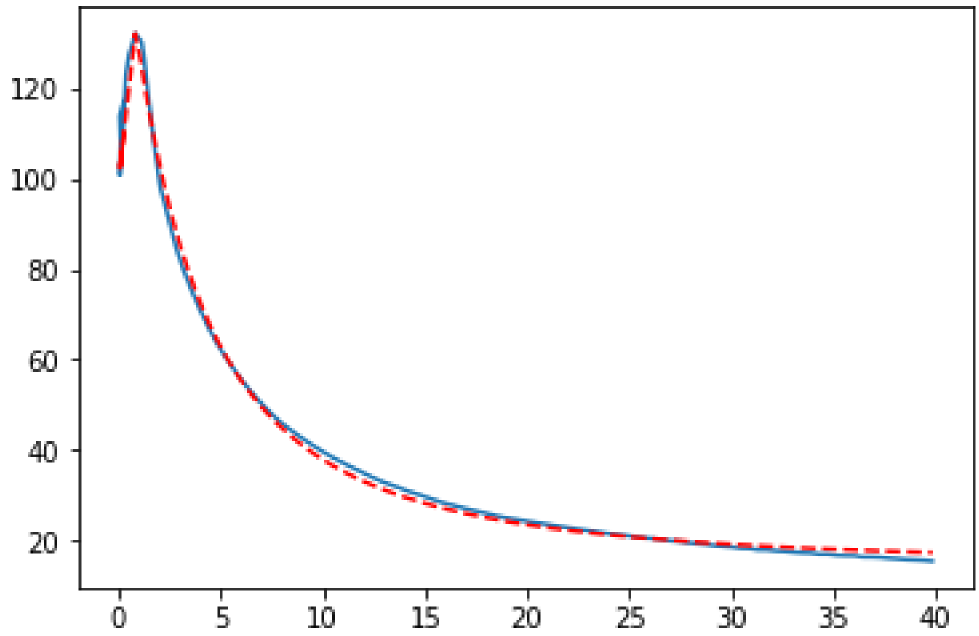
Out[9]: <function matplotlib.pyplot.show>



In [10]:
```
popt1x, pcov1x = curve_fit(pw1x, x, y_obs, p0 = [100,2,15])
```

In [11]:
```
plt.plot(x, y_obs)
plt.plot(x, pw1x(x,popt1x[0],popt1x[1],popt1x[2]), 'r--')
plt.show
```

Out[11]: <function matplotlib.pyplot.show>

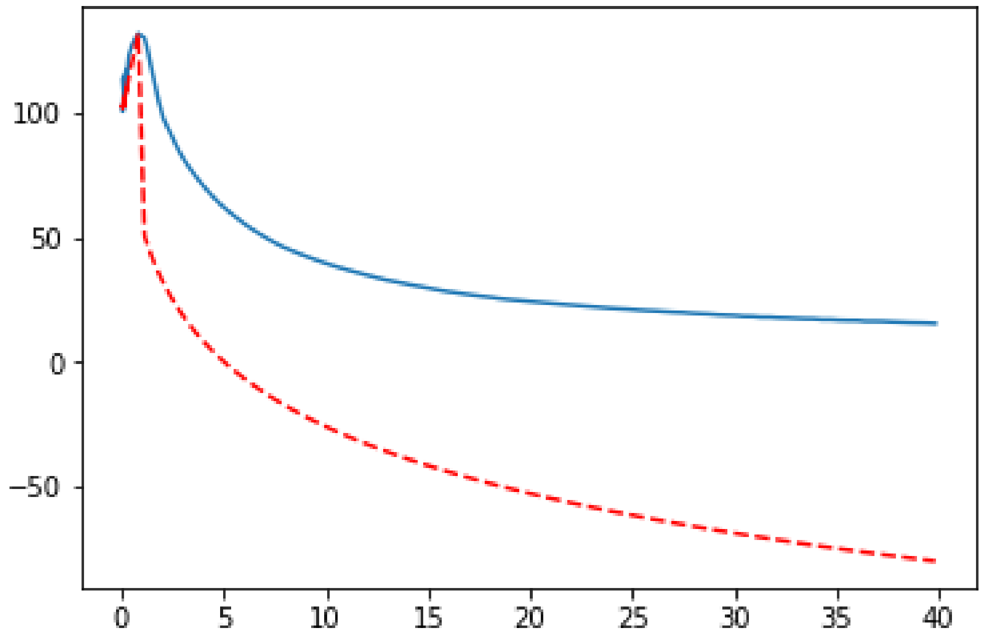


1.3 Define natural log function to fit the data

```
In [12]: def pwln(x, m2, m3, c2):
             y0 = 100
             y1 = 132.09
             a0 = 0
             a1 = 0.82

             m1 = (y1-y0)/(a1-a0)

             b2 = y1

             conds = [
                     (x >= a0) & (x <= a1),
                      x > a1
                     ]
             funcs = [
                     lambda x: y0 + m1*x,
                     lambda x: b2 + m2*np.log(m3*x - c2),
                     ]
             return np.piecewise(x, conds, funcs)
```

```
In [13]: plt.plot(x, y_obs)
         plt.plot(x, pwln(x, -40,5,-2), 'r--')
         plt.show
```

Out[13]: <function matplotlib.pyplot.show>

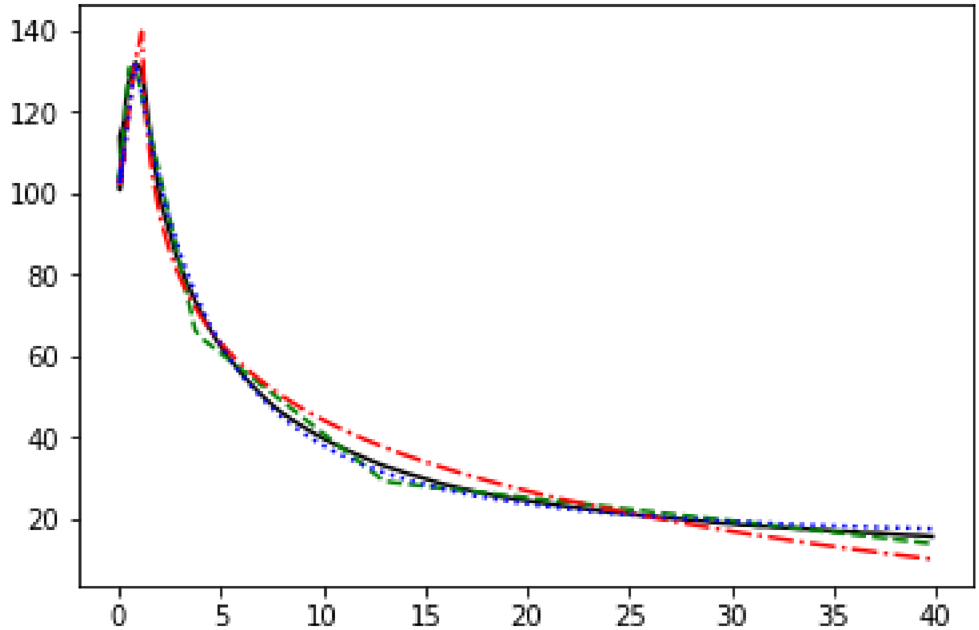



```
In [14]: poptln, pcovln = curve_fit(pwln, x, y_obs, p0 = [-40,1,-2])
```

1.4 Visually test all three functions

```
In [15]: plt.plot(x, y_obs, 'black')
         plt.plot(x, pwlin(x,poptlin[0],poptlin[1],poptlin[2],poptlin[3],poptlin[4],poptlin[5],p
         plt.plot(x, pwln(x, poptln[0], poptln[1], poptln[2]), 'r-.')
         plt.plot(x, pw1x(x, popt1x[0], popt1x[1], popt1x[2]), 'b:')
         plt.show
```

Out[15]: <function matplotlib.pyplot.show>



The quadratic function appears to perform best, which can be confirmed quantitatively:

```
In [16]: maelin = np.mean(abs(y_obs - pwlin(x,poptlin[0],poptlin[1],poptlin[2],poptlin[3],poptli
         maeln = np.mean(abs(y_obs - pwln(x, poptln[0], poptln[1], poptln[2])))
         mae1x = np.mean(abs(y_obs - pw1x(x, popt1x[0], popt1x[1], popt1x[2])))

         print('MAE for piecewise quadratic function: ', np.around(mae1x,2))
         print('MAE for piecewise linear function: ', np.around(maelin,2))
         print('MAE for piecewise natural log function: ', np.around(maeln,2))
```

```
MAE for piecewise quadratic function:  1.61
MAE for piecewise linear function:  1.75
MAE for piecewise natural log function:  3.3
```

The optimized parameters for the quadratic function are:

```
In [17]: popt1x
```

```
Out[17]: array([ 6375.15046863,     6.46889438,  -117.65100993])
```