

The new compaction equation in [Densification](#) paper was described as follows:

Below a thin surface layer of new snow, in which there is rapid metamorphism and settling, and for $\rho \leq 550 \text{ kg m}^{-3}$, we write

$$\dot{\epsilon} = \frac{k_0^*}{\rho_w g} \left(\frac{\rho_i - \rho}{\rho} \right) (1 - \bar{M}_0 m) \frac{1}{H(\tau)} \exp \left(-\frac{E_\alpha}{RT} \right) \sigma \quad (1)$$

where $H(\tau)$ is a “temperature-history function” defined by

$$H(\tau) = \int_{\tau_0}^{\tau} \exp \left(-\frac{E_\alpha}{RT(\tau')} \right) d\tau'. \quad (2)$$

Here ρ_i and ρ_w are the densities of ice and water, R is the gas constant, E_α is the apparent activation energy for compaction at temperature T and k_0^* is the densification constant. The time from deposition is τ , and the time at which the snow element leaves the thin surface layer is τ_0 .

This equation represents a simple conceptual model based on one densification process, with activation energy E_α . The strength of the snow decreases as current temperature increases, but increases with past exposure to higher temperatures. That is, we suppose that the rate-limiting process in developing strength is the same process that moves grains closer together, because it allows new bonds to form. One consequence of this choice is that the strength is related to the current density; the other is that the effect of exposure to temperature on the strength can be expressed by the temperature history function $H(\tau)$.

This description was based on an incorrect calculation of the history function. The corrected text describing this calculation reads as follows:

Using equation A1 to calculate $T(\tau)$ we find that the history function, $H(\tau)$, is virtually linear below the first annual layer, with a gradient that decreases with increasing \bar{a} . Figure 1(c) shows the history function for a typical activation energy $E_\alpha = 110 \text{ kJ mol}^{-1}$ at T41D. (The determination of E_α is discussed in detail in Section 6.3). The gradient of the linear fit to $H(\tau)$ is $1.4 \cdot 10^{-23} \text{ a}^{-1}$. Varying the accumulation over the range $0.02 - 0.6 \text{ m w.e. a}^{-1}$ at fixed mean annual temperature produces a variation in gradient of $(4.8 - 0.6) \cdot 10^{-23} \text{ a}^{-1}$.

We now make the approximation

$$\begin{aligned} H(\tau) &\approx \exp \left(-\frac{E_H}{RT_m} \right) \tau \\ &\approx -\exp \left(-\frac{E_H}{RT_m} \right) \frac{\bar{\sigma}}{\bar{a}\rho_w g} \end{aligned} \quad (3)$$

where E_H is an effective activation energy which will be close, but not equal, to E_α , and $\bar{\sigma}$, the time-invariant average stress at a given depth, is defined as $-\bar{a}g\rho_w\tau$. Equation (1) then becomes

$$\dot{\epsilon} \approx -\bar{a}k_0^* \left(\frac{\rho_i - \rho}{\rho} \right) (1 - \bar{M}_0 m) \exp \left(\frac{E_H}{RT_m} \right) \exp \left(-\frac{E_\alpha}{RT} \right) \frac{\sigma}{\bar{\sigma}} \quad (4)$$

For the deeper parts of the snowcover, where $\sigma \approx \bar{\sigma}$ and $T \rightarrow T_m$, equation (4) reduces to

$$\dot{\epsilon} \approx -\bar{a}k_0^* \left(\frac{\rho_i - \rho}{\rho} \right) (1 - \bar{M}_0 m) \exp \left(-\frac{(E_\alpha - E_H)}{RT_m} \right) \quad (5)$$

and there is no trend in the strain rate with increasing overburden. For $\rho = \rho_0$, $m = 0$ and equation (5) reduces further to

$$\dot{\epsilon} \approx -\bar{a}k_0^* \left(\frac{\rho_i - \rho}{\rho} \right) \exp \left(-\frac{(E_\alpha - E_H)}{RT_m} \right) \quad (6)$$

which has the same form as the purely empirical equation derived by *Herron and Langway* (1980) for snow with $\rho \leq 550 \text{ kg m}^{-3}$

$$\dot{\epsilon} = -\bar{a}k_0^* \left(\frac{\rho_i - \rho}{\rho} \right) \exp \left(-\frac{E^*}{RT_m} \right). \quad (7)$$

The *Herron and Langway* (1980) data give $k_0^* = 11 (+6, -4) \text{ m w.e.}^{-1}$ and $E^* = 10.16 \pm 0.94 \text{ kJ mol}^{-1}$.

The conclusion that this is close to the range of values of $E_\alpha - E_H$ shown in Table 1 for three polar locations with widely-differing climate conditions is now only true for $E_\alpha = 200 \text{ kJ mol}^{-1}$. Since the best value for the EGIG line data is in fact 112 kJ mol^{-1} it looks as if the hypothesis that there is only one process cannot hold. Another term in T_m is

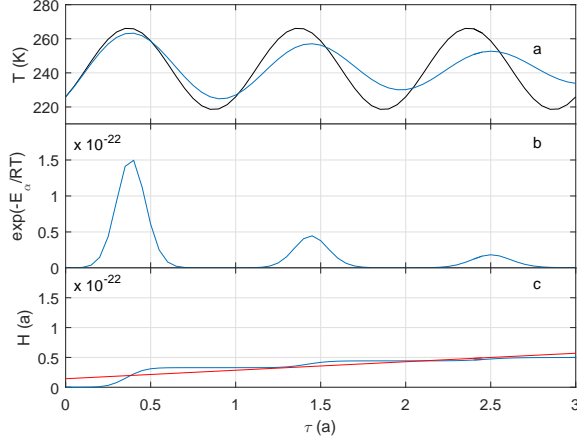


Figure 1: Estimated effect of the annual temperature variation at T41D where $\bar{a} = 0.22$ m w.e. a^{-1} and $T_m = 242.34$ K ($T_m^* = -30.8^\circ\text{C}$). (a) The temperature, T , at the surface (black) and for an element of snow deposited at time $\tau = 0$ as it moves downwards in the pack (blue). As the snow ages the amplitude of the temperature wave it experiences decreases. (b) The magnitude of the rate factor for a temperature-driven process with activation energy $E_\alpha = 110$ kJ mol^{-1} . This decreases more rapidly with time but is still significant for several years. (c) The temperature-history function $H(\tau)$ given $E_\alpha = 110$ kJ mol^{-1} (blue) and a linear fit (red).

Table 1: $(E_\alpha - E_H)$ in kJ mol^{-1} .

Site	T_m^* $^\circ\text{C}$	\bar{a} m w.e. a^{-1}	$E_\alpha = 60$ kJ mol^{-1}	$E_\alpha = 110$ kJ mol^{-1}	$E_\alpha = 200$ kJ mol^{-1}
South Pole	-51.0	0.07	2.7	6.5	13.9
T41D	-30.8	0.22	1.5	4.0	9.7
Roi Baudouin	-15.0	0.38	0.8	2.6	7.1

required. The easiest way to make the new equation correspond to *Herron and Langway (1980)* is by substituting k_0 for k_0^* in equation 1 where

$$k_0 = k_0^* \exp \left(-\frac{E^* - (E_\alpha - E_H)}{RT_m} \right) \quad (8)$$

Note that there is also an error in section 5.4 where the wrong value of $E_\alpha - E_H$ is used to calculate k_0^* . The lower curve in Figure 15 is not now relevant

References

- Alley, R. B. (1987), Firn densification by grain-boundary sliding: a first model, *Journal de Physique Colloque*, 3(48), 249–254.
- Arthern, R. J., and D. J. Wingham (1998), The natural fluctuations of firn densification and their effect on the geodetic determination of ice sheet mass balance, *Climate Change*, 40, 605–624.
- Arthern, R. J., D. G. Vaughan, A. M. Rankin, R. Mulvaney, and E. R. Thomas (2010), In-situ measurements of antarctic snow compaction compared with predictions of models., *Journal of Geophysical Research*, 115(F03011), 10.1029/2009JF001, 306.
- Chen, J. S., and C. W. Liu (2011), Generalized analytical solution for advection-dispersion equation in finite spatial domain with arbitrary time-dependent inlet boundary condition, *Hydrology and Earth System Sciences*, 15(8), 2471–2479.
- Coble, R. L. (1970), Diffusion models for hot pressing with surface energy and pressure effects as driving forces, *Journal of Applied Physics*, 41(12), 4798.
- Freitag, J., F. Wilhelms, and S. Kipfstuhl (2004), Microstructure-dependent densification of polar firn derived from x-ray microtomography, *Journal of Glaciology*, 50(169), 243–250.
- Fujita, S., J. Okuyama, A. Hori, and T. Hondoh (2009), Metamorphism of stratified firn at Dome Fuji, Antarctica: A mechanism for local insolation modulation of gas transport conditions during bubble close off, *Journal of Geophysical Research*, 114.

- Gerland, S., H. Oerter, J. Kipfstuhl, F. Wilhelms, H. Miller, and W. D. Miners (1999), Density log of a 181 metre long ice core from Berkner Island, Antarctica, *Annals of Glaciology*, 29, 215–219.
- Hawley, R. L., and E. D. Waddington (2011), In situ measurements of firn compaction profiles using borehole optical stratigraphy, *Journal of Glaciology*, 57(202), 289–294.
- Herron, M. M., and C. C. Langway (1980), Firn densification: an empirical model, *Journal of Glaciology*, 25(93), 373–385.
- Hoch, S. (2005), Radiative flux divergence in the surface boundary layer. A study based on observations at Summit, Greenland, Ph.D. thesis.
- Hörhold, M. W., S. Kipfstuhl, F. Wilhelms, J. Freitag, and A. Frenzel (2011), The densification of layered polar firn, *J. Geophys. Res.*, 116(F1), 1–15.
- Li, J., H. Zwally, H. Cornejo, and D. Yi (2003), Seasonal variation of snow-surface elevation in North Greenland as modeled and detected by satellite radar altimeter, *Annals of Glaciology*, 37, 233–238.
- Maeno, N., and T. Ebinuma (1983), Pressure sintering of ice and its implication to the densification of snow at polar glaciers and ice sheets, *Journal of Physical Chemistry*, 87, 4103–4110.
- Morris, E., and D. J. Wingham (2011), The effect of fluctuations in surface density, accumulation and compaction on elevation change rates along the EGIG line, central Greenland, *Journal of Glaciology*, 57(203), 416–430.
- Morris, E. M. (2008), A theoretical analysis of the neutron scattering method of measuring snow and ice density, *Journal of Geophysical Research*, 113(F03019), doi:10.1029/2007JF000, 962.
- Morris, E. M., and J. D. Cooper (2003), Density measurements in ice boreholes using neutron scattering, *Journal of Glaciology*, 49(167), 599–604.
- Reeh, N., D. A. Fisher, R. M. Koerner, and H. B. Clausen (2005), An empirical firn-densification model comprising ice lenses, *Annals of Glaciology*, 42, 101–106.
- Robin, G. d. Q. (1958), Glaciology. III. Seismic shooting and related investigations., in *Norwegian-British-Swedish Antarctic Expedition, 1949-52. Scientific Results*, vol. 5.
- Scapozza, C., and P. A. Bartelt (2003), The influence of temperature on the small-strain viscous deformation mechanics of snow: a comparison with polycrystalline ice, *Annals of Glaciology*, 37, 90–96.
- Spencer, M., R. B. Alley, and T. Creyts (2001), Preliminary firn-densification model with 38-site dataset, *Journal of Glaciology*, 47(159), 671–676.
- Steffen, K., and J. Box (2001), Surface climatology of the Greenland ice sheet: Greenland climate network 1995-1999, *Journal of Geophysical Research*, 106(D24), 33,951–33,964.
- Wilkinson, D. S. (1988), Pressure sintering model for the densification of polar firn and glacier ice, *Journal of Glaciology*, 34(116), 40–45.
- Yen, Y.-C. (1981), Review of thermal properties of snow, ice and sea ice, *US Army Cold Regions Research and Engineering Laboratory Report*, 81-10.
- Zwally, H. J., and J. Li (2002), Seasonal and interannual variations of firn densification and ice-sheet elevation at the Greenland summit, *Journal of Glaciology*, 48(161), 199–207.